

Perturbations of the metric induced by back - reaction in the warm inflation scenario

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Abstract

A second - order expansion for the quantum fluctuations of the matter field was considered in the framework of the warm inflation scenario. The friction and Hubble parameters were expanded by means of a semiclassical approach. The fluctuations of the Hubble parameter generates fluctuations of the metric. These metric fluctuations produce an effective term of curvature. The power spectrum for the metric fluctuations can be calculated on the infrared sector.

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I. INTRODUCTION

Since its early developments, quantum fluctuations has played an important role in the inflationary universe [1–6]. They lead to cosmological density perturbations that may be responsible for the origin of structures in the universe [7] and may completely alter our concepts about the past, the future, and global structure of spacetime [8]. A promising approach towards a better understanding of these phenomena is the paradigm of stochastic inflation [9]. The aim of stochastic inflation is to include the quantum contributions in an effective classical theory, where they appear as stochastic noises.

A natural consequence of this approach is the self - reproduction of universes and the return to a global stationary picture. The period in which $\ddot{a} > 0$ and thus the universe has an accelerated expansion is the inflationary stage, and models are discarded or not depending on the fact that they provide enough inflation or not. The standard inflationary model separates expansion and reheating as two distinguished time periods. This theory assumes

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an exponential expansion in a second - order phase transition of the inflaton field [10], followed by localized mechanism that rapidly distributes the vacuum energy into thermal energy. Reheating after inflation occurs due to particle production by the oscillating inflaton field [11]. The differential microwave radiometer (DMR) on the Cosmic Background Explorer (COBE) has made the first direct probe of the initial density perturbations through detection of the temperature anisotropies in the cosmic background radiation (CBR). The results are consistent with the scaling spectrum given by the inflation model. For inflation the simplest assumption is that there are two scales: a long - time, long - distance scale associated with vacuum energy dynamics and a single short - time, short - distance scale associated with a random force component. The Hubble time during inflation, $1/H$, appropriately separates the two regimes. For the grand unified theory [12], this time interval is $1/H \sim 10^{-34}$ sec. Inflation predicts that the initial density perturbations should be Gaussian, with a power - law spectrum index $n \sim 1$ [13]. Furthermore, the existence of nucleation of matter in the universe should be a consequence of these early perturbations of the matter field.

The warm inflation scenario takes into account separately, the matter and radiation energy fluctuations. This formalism, that mixes these two isolated exponential expansion and reheating stages, may solve the disparities created by the two each periods. In this scenario introduced by Berera and Fang [14,15], the thermal fluctuations could play the dominant role in producing the initial perturbations. The warm inflation scenario served as a explicit demonstration that inflation can occur in the presence of a thermal component.

In an alternative formalism for warm inflation [16,17] I demonstrate that, for a potential inflation model both, thermal equilibrium and quantum to classical transition of the coarse - grained field hold for a sufficiently large rate of expansion of the scale factor of the universe. For a power - law expanding for scale factor, the mean temperature decreases as $t^{-1/2}$. So, one can predicts the dynamics for the mean temperature and the amplitude of their fluctuations. Thus, at the end of the inflationary era the thermal equilibrium holds and the spectrum of the coarse - grained matter field can be calculated.

Any local energy perturbations during inflation can affect a region of characteristic physical length $1/H(\varphi)$ or less. The largest scales of energy density fluctuations in the post - inflationary universe arose from the earliest perturbations during inflation. For inflation we understand naturality as both macroscopic and microscopic. Macroscopically, we would like a description that rests with common - day experience. Microscopically, it should be consistent with the standard model of particle physics. The main drawback of these approaches (i.e., the standard and warm inflation ones) is the slow - roll assumption itself, which gives the reduction of the equation of motion of the scalar field to a first - order one.

In this work I develop a formalism where the quantum field of matter φ interacts with other fields of a thermal bath at mean temperature $\langle T_r \rangle < T_{GUT} \sim 10^{15}$ GeV. This lower temperature condition implies that magnetic monopole suppression works effectively. In this model quantum fluctuations of the matter field φ lead to quantum fluctuations of the metric in addition to matter and radiation energy densities. I consider a general case, where the inflaton field has a nonzero mean value and I develop the analysis by using a consistent semiclassical expansion. This work is organized as follow: In section II) I develop the formalism for a quantum perturbed flat FRW metric. Classical and quantum dynamics are studied with a semiclassical approach on a flat FRW background metric given by the expectation value of the perturbed flat FRW metric. In section III) I study the coarse -

grained approach for the matter and metric fluctuations to warm inflation. The expression for the power spectrum of the metric fluctuations is obtained. Finally, in section IV) some final remarks are developed.

II. FORMALISM

In the warm inflation era, the radiation energy density must be small with respect to the vacuum energy, which is given by the potential energy density $V(\varphi)$. Furthermore the kinetic component of the energy density is negligible with respect to the vacuum energy density

$$\rho(\varphi) \sim \rho_m \sim V(\varphi) \gg \rho_{kinetic}.$$

where $\rho_{kinetic} = \rho_r(\varphi) + \frac{1}{2}\dot{\varphi}^2$ and $\rho_r(\varphi) = \frac{\tau(\varphi)}{8H(\varphi)}\dot{\varphi}^2$. Here, $H(\varphi)$ and $\tau(\varphi)$ are the Hubble and friction parameters. The conventional treatment for the scalar field dynamics assumes that it is pure vacuum energy dominated. The various kinematic outcomes are a result of specially chosen Lagrangians. In most cases the Lagrangian is unmotivated from particle phenomenology. Clear exceptions are the Coleman - Weinberg potential with a coupling constant, which is motivated by grand unified theories and supersymmetric potentials [12]. Making an extension to the new inflation picture, the behavior of the scale factor can also be altered for any given potential when radiation energy is present. In our case the density Lagrangian that describes the warm inflation scenario is

$$\mathcal{L}(\varphi, \varphi_{,\mu}) = -\sqrt{-g} \left[\frac{R}{16\pi} + \frac{1}{2}g^{\mu\nu}\varphi_{,\mu}\varphi_{,\nu} + V(\varphi) \right] + \mathcal{L}_{int}. \quad (1)$$

Here, R is the scalar curvature, $g^{\mu\nu}$ the metric tensor, g is the metric. The Lagrangian \mathcal{L}_{int} takes into account the interaction of the field φ with other particles in the thermal bath. All particlelike matter which existed before inflation would have been dispersed by inflation.

As in previous works [16–19], I consider a semiclassical approach for a quantum operator $\varphi(\vec{x}, t)$

$$\varphi(\vec{x}, t) = \phi_c(t) + \phi(\vec{x}, t), \quad (2)$$

where $\langle E|\varphi|E \rangle = \phi_c(t)$ is the expectation value of the operator φ in an arbitrary state $|E \rangle$. Furthermore, I require that $\langle E|\dot{\phi}|E \rangle = 0$ and $\langle E|\phi(\vec{x}, t)|E \rangle = 0$.

One can write the perturbed Hubble parameter as an expansion in ϕ

$$H(\varphi) = H_c(\phi_c) + \sum_{n=1}^{\infty} \frac{1}{n!} H^{(n)}(\phi_c) \phi^n, \quad (3)$$

where $H_c(\phi_c) \equiv H(\phi_c)$ and $H^{(n)}(\phi_c) \equiv \frac{d^n H(\varphi)}{d\varphi^n} \Big|_{\phi_c}$. I will consider the quantum fluctuations ϕ as very small. Thus, will be sufficient to consider a ϕ - first - order expansion in eq. (3).

Now we consider a perturbed flat Friedmann - Robertson - Walker (FRW) metric

$$ds^2 = -dt^2 + a^2(t) [1 + h(\vec{x}, t)] d\vec{x}^2, \quad (4)$$

where $a(t) = a_o e^{\int H_c(\phi_c) dt}$ is the classical scale factor of the universe and $h(\vec{x}, t)$ represents the quantum fluctuations of the metric, such that

$$1 + h(\vec{x}, t) = e^{2 \int H'(\phi_c) \phi(\vec{x}, t) dt}. \quad (5)$$

When the fluctuations $\phi(\vec{x}, t)$ are very small the field $h(\vec{x}, t)$ can be approximated to

$$h(\vec{x}, t) \simeq 2 \int H'(\phi_c) \phi(\vec{x}, t) dt. \quad (6)$$

Note that $\langle E | h(\vec{x}, t) | E \rangle = 0$ and thus

$$\langle E | ds^2 | E \rangle = -dt^2 + a^2(t) d\vec{x}^2, \quad (7)$$

which gives a globally flat FRW metric. Then, the expectation value of the metric $\langle E | ds^2 | E \rangle$ gives the background metric.

The quantum equation of motion for the operator φ in a globally flat FRW spacetime is

$$\ddot{\varphi} - \frac{1}{a^2(t)} \nabla^2 \varphi + [3H(\varphi) + \tau(\varphi)] \dot{\varphi} + V'(\varphi) = 0. \quad (8)$$

The expression (8) with $\tau(\varphi) = 0$ gives the equation of motion for standard inflation [19]. In this work I will study the case where $\tau(\varphi) \neq 0$. As the inflation relaxes toward its minimum energy configuration, it will decay into lighter fields, generating an effective viscosity [20]. If this viscosity is large enough, the inflaton will reach a slow - roll regime, where its dynamics becomes overdamped. This overdamped regime has been analyzed in Ref. [21].

The semiclassical Friedmann equation for a globally flat FRW metric is

$$\langle E | H^2(\varphi) | E \rangle = \frac{8\pi}{3M_p^2} \langle E | \rho_m + \rho_r | E \rangle, \quad (9)$$

where $M_p = 1.2 \cdot 10^{19}$ GeV is the Planckian mass. Here, the matter and radiation energy densities, $\rho_m(\varphi)$ and $\rho_r(\varphi)$, are

$$\rho_m(\varphi) = \frac{\dot{\varphi}^2}{2} + \frac{1}{2a^2} (\vec{\nabla} \varphi)^2 + V(\varphi), \quad (10)$$

$$\rho_r(\varphi) = \frac{\tau(\varphi)}{8H(\varphi)} \dot{\varphi}^2. \quad (11)$$

In the limiting case of standard inflation [$\tau(\varphi) = 0$], the radiation energy density becomes zero.

We can write $V'(\varphi)$ and the friction parameter $\tau(\varphi)$ in eq. (8), as ϕ - expansions

$$V'(\varphi) = V'(\phi_c) + \sum_{n=1}^{\infty} \frac{1}{n!} V^{(n+1)}(\phi_c) \phi^n, \quad (12)$$

$$\tau(\varphi) = \tau_c + \sum_{n=1}^{\infty} \frac{1}{n!} \tau^{(n)}(\phi_c) \phi^n. \quad (13)$$

Replacing $V'(\varphi)$ (at second order in ϕ) with $H(\varphi)$, $\tau(\varphi)$ and φ (at first order in ϕ) in eqs. (8) and (9), we obtain the following motion and Friedmann equations

$$\ddot{\phi} + \ddot{\phi}_c - \frac{1}{a^2(t)} \nabla^2(\phi_c + \phi) + [3(H_c + H'\phi) + (\tau_c + \tau'\phi)](\dot{\phi}_c + \dot{\phi}) + V'(\phi_c) + V''(\phi_c)\phi + \frac{1}{2}V'''(\phi_c)\phi^2 = 0, \quad (14)$$

and

$$\begin{aligned} \langle E | (H_c + H'\phi)^2 | E \rangle &= \frac{4\pi}{3M_p^2} \left\langle E \left| (\dot{\phi}_c + \dot{\phi})^2 \left[1 + \frac{\tau_c + \tau'\phi}{4(H_c + H'\phi)} \right] + \frac{1}{a^2(t)} [\vec{\nabla}(\phi_c + \phi)]^2 \right. \right. \\ &\quad \left. \left. + 2 \left[V(\phi_c) + V'(\phi_c)\phi + \frac{1}{2}V''(\phi_c)\phi^2 \right] \right| E \right\rangle, \end{aligned} \quad (15)$$

where we have expanded the Hubble and friction parameters $[H(\varphi)$ and $\tau(\varphi)]$, at first order in ϕ . Furthermore, $V(\varphi)$ and $V'(\varphi)$ were expanded at second order in ϕ . The friction parameter takes into account the interaction of the matter field with the fields in the thermal bath. The eq. (14) describes the dynamics for a semiclassical expansion of the matter field φ . The eq. (15) is the second order semiclassical approach for the Friedmann equation with this expansion for φ .

A. Dynamics of the classical field

We consider the eq. (14) at zero order in ϕ . The equation of motion for the field $\phi_c(t)$ is

$$\ddot{\phi}_c + [3H_c(\phi_c) + \tau_c(\phi_c)]\dot{\phi}_c + V'(\phi_c) = 0. \quad (16)$$

Note that making $\tau_c = 0$, $|\ddot{\phi}_c| \ll 3H_c(\phi_c)|\dot{\phi}_c|$ and $|\ddot{\phi}_c| \ll V'(\phi_c)$ one obtains the limit case for slow - roll regime in standard inflation. Furthermore, for $\tau_c(\phi_c) \gg 3H_c(\phi_c)$ (i.e., for $\gamma \gg 1$), $|\ddot{\phi}_c| \ll \tau_c(\phi_c)|\dot{\phi}_c|$ and $|\ddot{\phi}_c| \ll V''(\phi_c)$, one recover the slow - roll limiting regime in warm inflation. The dynamics for both, the classical field ϕ_c and the classical Hubble parameter $H_c(\phi_c)$, are

$$\dot{\phi}_c = -\frac{M_p^2}{4\pi} H'_c \left(1 + \frac{\tau_c}{3H_c} \right)^{-1}, \quad (17)$$

$$\dot{H}_c = H'_c \dot{\phi}_c = -\frac{M_p^2}{4\pi} (H'_c)^2 \left(1 + \frac{\tau_c}{3H_c} \right)^{-1}. \quad (18)$$

Observe that $\dot{H}_c < 0$, which means that the classical Hubble parameter decreases with time. Equations (17) and (18) define the classical evolution of the spacetime (i.e., the background spacetime). The eq. (15) at zero order in ϕ , gives the classical Friedmann equation

$$H_c^2(\phi_c) = \frac{4\pi}{3M_p^2} \left[\left(1 + \frac{\tau_c}{4H_c} \right) \dot{\phi}_c^2 + 2V(\phi_c) \right]. \quad (19)$$

From eqs. (17) and (19), the classical potential becomes

$$V(\phi_c) = \frac{3M_p^2}{8\pi} \left[H_c^2(\phi_c) - \frac{M_p^2}{12\pi} (H'_c)^2 \left(1 + \frac{\tau_c}{4H_c} \right) \left(1 + \frac{\tau_c}{3H_c} \right)^{-2} \right]. \quad (20)$$

The effective classical potential depends on both, the friction and Hubble parameters. The classical expressions the matter and radiation energy densities, are

$$\rho_m(\phi_c) = \frac{1}{2} \left[\dot{\phi}_c^2 + 2V(\phi_c) \right], \quad (21)$$

$$\rho_r(\phi_c) = \frac{\tau_c}{8H_c} \dot{\phi}_c^2. \quad (22)$$

The characteristic time scale for inflation is $\tau_H = 1/H_c$, and the number of e - folds in inflation is

$$N_e = \int_{t_o}^{t_{end}} \frac{dt}{\tau_H}. \quad (23)$$

The basic idea of warm inflation is quite simple. A scalar field coupled to several fields in a thermal bath. As the inflaton relaxes toward its minimum energy configuration, it will decay into lighter fields, generating an effective friction. If this friction is large enough, the inflation will reach a slow - roll regime, where its dynamics becomes overdamped. In order to satisfy one of the requirements of a successful inflation (60 or more e-folds), overdamping must be very efficient [15].

In the next subsections I will develop the dynamics for the quantum field ϕ which gives information about the spatial inhomogeneities of matter in the universe.

B. first - order Dynamics of the quantum perturbations

From eq. (14) one obtains the equation of motion for a ϕ - first - order expansion

$$\ddot{\phi} - \frac{1}{a^2(t)} \nabla^2 \phi + 3H_c \left(1 + \frac{\tau_c}{3H_c} \right) \dot{\phi} + \left[3H' \left(1 + \frac{\tau'}{3H'} \right) \dot{\phi}_c + V'' \right] \phi = 0. \quad (24)$$

The Friedmann equation becomes [see eq. (15)]

$$\langle E | 2H_c H' \phi | E \rangle = \frac{4\pi}{3M_p^2} \left\langle E \left| \dot{\phi}_c \left[\frac{\dot{\phi}_c}{4H_c} \left[\tau' - \frac{H' \tau_c}{4H_c} \right] \phi + 2 \left(1 + \frac{\tau_c}{4H_c} \right) \dot{\phi} \right] + 2V'(\phi_c) \phi \right| E \right\rangle, \quad (25)$$

which is zero, due to $\langle E | \phi(\vec{x}, t) | E \rangle = \langle E | \dot{\phi}(\vec{x}, t) | E \rangle = 0$. Furthermore, from eq. (17) one obtains the equation of motion for the matter field fluctuations ϕ

$$\ddot{\phi} - \frac{1}{a^2(t)} \nabla^2 \phi + [3H_c + \tau_c] \dot{\phi} + \left[V''(\phi_c) - \frac{3M_p^2}{4\pi} (H')^2 \left(1 + \frac{\tau'}{3H'} \right) \left(1 + \frac{\tau_c}{3H_c} \right)^{-1} \right] \phi = 0, \quad (26)$$

where I have used $H' H'_c = (H')^2$.

We can introduce the new field $\chi = e^{3/2 \int (H_c + \tau_c/3) dt} \phi$ to describe the quantum fluctuations of the matter field. The eq. (26), written as a function of χ , is

$$\ddot{\chi} - \frac{1}{a^2} \nabla^2 \chi - \frac{k_o^2(t)}{a^2(t)} \chi = 0, \quad (27)$$

with

$$k_o^2(t) = a^2(t) \left[\frac{3M_p^2}{4\pi} (H')^2 \left(1 + \frac{\tau_c}{3H_c} \right)^{-1} \left(1 + \frac{\tau'}{3H'} \right) + \frac{9}{4} \left(H_c + \frac{\tau_c}{3} \right)^2 + \frac{3}{2} \left(\dot{H}_c + \frac{\dot{\tau}_c}{3} \right) - V''(\phi_c) \right] \quad (28)$$

The eq. (27) is a Klein - Gordon equation for χ in a globally flat FRW background metric, with a time - dependent parameter of mass $\mu(t) = \frac{k_o}{a}$. Note that k_o depends with t [see eq. (28)]. The very important difference between k_o in eq. (28) and other approximations is that here there appears a new term $\left(1 + \frac{\tau_c}{3H_c} \right)^{-1} \frac{\tau'}{3H'}$ due to the fact that we have considering the fluctuations for the Hubble and friction parameters.

Now we define the quantum perturbations χ , as a Fourier expansion in terms of the modes $\xi_k(t)e^{i\vec{k}\cdot\vec{x}}$

$$\chi(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left[a_k \xi_k(t) e^{i\vec{k}\cdot\vec{x}} + H.c \right], \quad (29)$$

where a_k^\dagger and a_k are the creation and annihilation operators

$$[a_k, a_{k'}^\dagger] = \delta^{(3)}(\vec{k} - \vec{k}'); \quad [a_k, a_{k'}] = [a_k^\dagger, a_{k'}^\dagger] = 0. \quad (30)$$

From eq. (6), the field $h(\vec{x}, t)$ becomes

$$h(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \left[a_k \tilde{\xi}_k(t) e^{i\vec{k}\cdot\vec{x}} + H.c \right], \quad (31)$$

which is the field that describes the quantum fluctuations of the metric ds^2 , written as a Fourier expansion in terms of the modes $\tilde{\xi}_k(t)e^{i\vec{k}\cdot\vec{x}}$. The time - dependent modes $\tilde{\xi}_k(t)$ are

$$\tilde{\xi}_k(t) = 2 \int H'(t) \xi_k(t) e^{-3/2 \int^t (H_c + \tau_c/3) dt'} dt. \quad (32)$$

The commutation relation between χ and $\dot{\chi}$ is

$$[\chi(\vec{x}, t), \dot{\chi}(\vec{x}', t)] = i \delta^{(3)}(\vec{x} - \vec{x}'), \quad (33)$$

which is valid for the following relation of the time - dependent modes $\xi_k(t)$:

$$\xi_k \dot{\xi}_k^* - \dot{\xi}_k \xi_k^* = i. \quad (34)$$

Hence, the commutation relation between $h(\vec{x}, t)$ and $\dot{h}(\vec{x}, t)$ is

$$[h(\vec{x}, t), \dot{h}(\vec{x}, t)] = \frac{1}{(2\pi)^3} \int d^3k (\tilde{\xi}_k \dot{\xi}_k^* - \dot{\tilde{\xi}}_k \tilde{\xi}_k^*) e^{-i\vec{k}\cdot(\vec{x}-\vec{x}')}, \quad (35)$$

which can be calculated once one obtains the modes ξ_k and the Hubble parameter $H_c(t) = \frac{\dot{a}(t)}{a(t)}$. Furthermore, the equation of motion for the time - dependent modes $\xi_k(t)$ is

$$\ddot{\xi}_k(t) + a^{-2}(t) [k^2 - k_o^2(t)] \xi_k(t) = 0, \quad (36)$$

for $k_o(t)$ given by eq. (28). Equation (36) is the same that the equation of an harmonic oscillator with square time - dependent frequency $\omega_k^2 = a^{-2}(t) [k^2 - k_o^2(t)]$.

C. second - order perturbations for the matter field

The dynamics of the quantum fluctuations for the matter field at first - order in ϕ was studied with detail in the last subsection. In this subsection I will develop the basic statements for perturbations of the matter field at second - order in ϕ .

Equating the terms at second - order in ϕ in eq. (14), one obtains

$$(3H' + \tau') \phi \dot{\phi} + 1/2 V''' \phi^2 = 0, \quad (37)$$

and the Friedmann eq. (15), at second - order in ϕ , is

$$\begin{aligned} \langle E | (H')^2 \phi^2 | E \rangle &\simeq \frac{4\pi}{3M_p^2} \left\langle E \left| \dot{\phi}^2 \left(1 + \frac{\tau_c}{4H_c} \right) + \frac{1}{a^2(t)} [\vec{\nabla} \phi]^2 \right. \right. \\ &+ \left\{ V''(\phi_c) + \frac{M_p^2}{4\pi} H'_c \left(1 + \frac{\tau_c}{3H_c} \right)^{-1} \right. \\ &\times \left. \left[\frac{1}{4H_c} \left(\tau' - \frac{H'\tau_c}{4H_c} \right) \frac{V'''}{(3H' + \tau')} - \frac{M_p^2}{4\pi} H'_c \left(1 + \frac{\tau_c}{3H_c} \right)^{-1} \frac{\tau' H'}{(4H_c)^2} \right] \right\} \phi^2 \left. \right| E \rangle, \end{aligned} \quad (38)$$

where the approximation $\frac{1}{4H_c[1+H'\phi/(4H_c)]} \simeq \frac{1}{4H_c} \left(1 - \frac{H'\phi}{4H_c} \right)$ was used. Due to $\dot{h}(\vec{x}, t) = 2H'\phi(\vec{x}, t)$, the eq. (38) can be written as

$$\begin{aligned} \left\langle E \left| \frac{(\dot{h})^2}{4} \right| E \right\rangle &\simeq \frac{4\pi}{3M_p^2} \left\langle E \left| \dot{\phi}^2 \left(1 + \frac{\tau_c}{4H_c} \right) + \frac{1}{a^2} (\vec{\nabla} \phi)^2 \right. \right. \\ &+ \left\{ V''(\phi_c) + \frac{M_p^2}{4\pi} H'_c \left(1 + \frac{\tau_c}{3H_c} \right)^{-1} \right. \\ &\times \left. \left[\frac{1}{4H_c} \left(\tau' - \frac{H'\tau_c}{4H_c} \right) \frac{V'''}{(3H' + \tau')} - \frac{M_p^2}{4\pi} H'_c \left(1 + \frac{\tau_c}{3H_c} \right)^{-1} \frac{\tau' H'}{(4H_c)^2} \right] \right\} \phi^2 \left. \right| E \rangle, \end{aligned} \quad (39)$$

which gives the effective curvature of the spacetime due to the fluctuations of the matter field. Thus, the term $\left\langle E \left| \frac{(\dot{h})^2}{4} \right| E \right\rangle = \frac{K}{a^2}$ gives an additional contribution in the semiclassical Friedmann equation, such that the eq. (15) becomes

$$\begin{aligned} H_c^2(\phi_c) + \frac{K}{a^2} &= \frac{8\pi}{3M_p^2} V(\phi_c) + \frac{M_p^2}{12\pi} (H')^2 \left(1 + \frac{\tau_c}{4H_c} \right) \left(1 + \frac{\tau_c}{3H_c} \right)^{-2} \\ &+ \frac{8\pi}{3M_p^2} \left\langle E \left| \dot{\phi}^2 \left(1 + \frac{\tau_c}{4H_c} \right) + \frac{1}{a^2} (\vec{\nabla} \phi)^2 + \left\{ V''(\phi_c) + \frac{M_p^2}{4\pi} H'_c \left(1 + \frac{\tau_c}{3H_c} \right)^{-1} \right. \right. \right. \\ &\times \left. \left[\frac{1}{4H_c} \left(\tau' - \frac{H'\tau_c}{4H_c} \right) \frac{V'''}{(3H' + \tau')} - \frac{M_p^2}{4\pi} H'_c \left(1 + \frac{\tau_c}{3H_c} \right)^{-1} \frac{\tau' H'}{(4H_c)^2} \right] \right\} \phi^2 \left. \right| E \rangle. \end{aligned} \quad (40)$$

Hence, the second - order fluctuations of the matter field introduces an additional curvature in the background spacetime $\langle E | ds^2 | E \rangle = -dt^2 + a^2(t) d\vec{x}^2$. The expression (40) can be written as

$$H_c^2(\phi_c) + \frac{K}{a^2} = \frac{8\pi}{3M_p^2} \langle E | \rho_m + \rho_r | E \rangle. \quad (41)$$

Here, $\langle E|\rho_m|E \rangle$ and $\langle E|\rho_r|E \rangle$, are the effective matter and radiation energy densities, which include the second - order fluctuations of the matter field. The mean temperature of the bath is given by

$$\langle T_r \rangle \propto [\langle E|\rho_r|E \rangle]^{1/4}. \quad (42)$$

III. THE COARSE - GRAINED FIELDS

To develop a stochastic treatment for quantum fluctuations of the matter field we must study the universe on a scale much greater than the scale of the observable universe. Thus, I consider the redefined quantum fluctuations χ as two pieces

$$\chi = \chi_{cg} + \chi_S. \quad (43)$$

The piece χ_{cg} takes into account only the modes with wavelength of size

$$l \geq \frac{1}{\epsilon k_o},$$

which is much bigger than the size of the horizon (with size $1/k_o$). This coarse - grained field is given by

$$\chi_{cg}(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \theta(\epsilon k_o - k) \left[a_k e^{i\vec{k} \cdot \vec{x}} \xi_k + H.c. \right], \quad (44)$$

where $\theta(\epsilon k_o - k)$ is a Heaviside function which acts as a suppression factor, and $\epsilon \ll 1$ is a constant. Moreover, one can define the coarse - grained field that represent the fluctuations of the metric on the infrared sector

$$h_{cg}(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \theta(\epsilon k_o - k) \left[a_k e^{i\vec{k} \cdot \vec{x}} \tilde{\xi}_k(t) + H.c. \right], \quad (45)$$

where $h = h_{cg} + h_S$. On the other hand, the pieces χ_S and h_S take into account the modes with wavenumbers greater than ϵk_o

$$\chi_S(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \theta(k - \epsilon k_o) \left[a_k e^{i\vec{k} \cdot \vec{x}} \xi_k + H.c. \right], \quad (46)$$

$$h_S(\vec{x}, t) = \frac{1}{(2\pi)^{3/2}} \int d^3k \theta(k - \epsilon k_o) \left[a_k e^{i\vec{k} \cdot \vec{x}} \tilde{\xi}_k(t) + H.c. \right]. \quad (47)$$

The quantum fluctuations with wavenumbers greater than ϵk_o are generally interpreted as inhomogeneities superimposed to the classical field. They are responsible for the density inhomogeneities generated during the inflation. The modes with $k/k_o > \epsilon$ are referred to as outside the superhorizon.

A. Quantum to Classical transition for the coarse - grained field

Quantum to classical transition of the quantum fluctuations ϕ occurs when the commutators (33) and (35) are zero. When this transition holds one obtains $\xi_k \dot{\xi}_k^* - \dot{\xi}_k \xi_k^* \simeq 0$. We can represent the modes by

$$\xi_k(t) = u_k(t) + i v_k(t), \quad (48)$$

where $u_k(t)$ and $v_k(t)$ are time - dependent real functions. The condition to obtain the complex to real transition of a given mode ξ_k is

$$\left| \frac{v_k(t)}{u_k(t)} \right| \ll 1. \quad (49)$$

The quantum to classical transition function (QCTF) $\alpha_k(t) = \left| \frac{v_k(t)}{u_k(t)} \right|$ was defined in previous works [17]. The modes are real when this function becomes nearly zero. The condition for the coarse-grained fields, $\chi_{cg}(\vec{x}, t)$ and $h_{cg}(\vec{x}, t)$, become classical when the modes with $k < \epsilon k_o$ become real. More exactly

$$\frac{1}{N(t)} \sum_{k=0}^{k=\epsilon k_o} \alpha_k(t) \ll 1, \quad (50)$$

where $N(t)$ is time - dependent number of degrees of freedom of the infrared sector. During inflation $N(t)$ increases with time, since $k_o(t)$ is also increasing with time. Hence, constantly new degrees of freedom with a given $k \ll k_o$ enters in the infrared sector. Quantum to classical transition for the coarse-grained fields, χ_{cg} and h_{cg} , occurs when $\alpha_{k=\epsilon k_o}(t) \ll 1$. It can be seen that the solutions of eq. (36) for $k < k_o$ are real, while for $k > k_o$ the time - dependent modes are complex. Hence, when (49) is satisfied by all the modes which were much bigger than the size of the horizon, the time - dependent modes ξ_k become real in the infrared sector. This condition impose restrictions on the vacuum. A similar condition than (49) also was obtained in the framework of standard inflation by D. Polarski and A. A. Starobinsky [22,23], but for $\alpha_k \gg 1$. However, both conditions (49) and the Polarski - Starobinsky's one, are equivalent.

B. Coarse - grained field fluctuations

When the quantum fluctuations become classical, the amplitude of the fluctuations of the field $\phi_{cg}^{(c)}$ are (in the following I denote $\langle E | \dots | E \rangle$ as $\langle \dots \rangle$) [24]

$$\langle [\phi_{cg}^{(c)}]^2(t) \rangle = \frac{e^{-3 \int^t (H_c + \tau_c/3) dt'}}{2\pi^2} \int_0^{\epsilon k_o} dk k^2 \xi_k^2(t). \quad (51)$$

The square fluctuations

$$\langle [h_{cg}^{(c)}(t)]^2 \rangle = \frac{1}{2\pi^2} \int_0^{\epsilon k_o} dk k^2 \tilde{\xi}_k^2(t), \quad (52)$$

give the temporal evolution for the amplitude for the fluctuations in the metric on the infrared sector. Note that $\tilde{\xi}_k(t)$ depends with τ_c [see eq. (32)]. Thus, the square fluctuations $\langle [h_{cg}^{(c)}]^2 \rangle$ are τ_c - dependent. To study the power spectrum $\mathcal{P}_{h_{cg}^{(c)}}(k)$ for the fluctuations of the metric, one can write the square fluctuations $\langle [h_{cg}^{(c)}]^2 \rangle$ as

$$\langle [h_{cg}^{(c)}(t)]^2 \rangle = \frac{1}{2\pi^2} \int_0^{\epsilon k_o(t)} \frac{dk}{k} \mathcal{P}_{h_{cg}^{(c)}}(k), \quad (53)$$

where the power spectrum for the fluctuations of the metric, $\mathcal{P}_{h_{cg}^{(c)}}(k)$, when the horizon entry is

$$\mathcal{P}_{h_{cg}^{(c)}}(k) = A(t_*) \left(\frac{k}{\epsilon k_o(t_*)} \right)^n f(k), \quad (54)$$

where t_* denotes the time when the horizon entry, for which $k_o(t_*) \simeq \pi H_o$ in comoving scale. Furthermore, the factor $A(t_*)$ gives the amplitude of the spectrum when the horizon entry, n is the spectral index, and $f(k)$ is the square - t_* - evaluated Heaviside function: $\theta^2(\epsilon k_o - k)$. The standard choice $n = 1$ gives a scale invariant spectrum. However, an experimentally constraint for the COBE data gives [25]

$$|n - 1.2| < 0.3.$$

IV. FINAL REMARKS

Warm inflation takes into account separately, the matter and radiation energy densities. During the inflationary era the mean temperature is smaller than the GUT one (i.e., $< T_r > < T_{GUT} \simeq 10^{15}$ GeV). This lower temperature condition implies that magnetic monopole suppression works effectively. In this work I developed a semiclassical formalism for warm inflation that takes into account the fluctuations of the metric around a flat FRW background metric. The fundamental aspects of the formalism here developed are the following:

1) I consider a semiclassical expansion for the field $\varphi = \phi_c + \phi$. In this framework, the Hubble and friction parameters were expanded at first order in ϕ . The semiclassical expansion for the Hubble parameter generates a second - order semiclassical Friedmann eq. (15). This equation takes into account the expectation value for both, matter and radiation energy densities. The radiation energy density is proportional to the mean temperature ($< \rho_r > \propto < T_r >^4$) of the thermal bath.

2) The equation of motion (24) is valid for a background metric which describes a globally flat FRW spacetime. However, this metric fluctuates due to the back - reaction with the fluctuations of the matter field. This metric fluctuations are described by the field $h(\vec{x}, t)$. The eq. (6) shows how $h(\vec{x}, t)$ depends with $\phi(\vec{x}, t)$. Due to the second - order ϕ - expansion in the semiclassical Friedmann eq. (15), an effective curvature K arises in the semiclassical Friedmann equation [see eq. (40)]. The effective curvature is related with the metric fluctuations by eq. (39).

3) The field $\phi(\vec{x}, t)$ can be written as a Fourier expansion. For this description, the relevant modes $\xi_k(t)$ describe the temporal evolution of ϕ . In this work I studied these perturbations on a scale much bigger than the size of the horizon. In the infrared sector, the temporal evolution for the square $h_{cg}^{(c)}$ - fluctuations are described by the modes $\tilde{\xi}_k$ [see eq. (52)]. In the infrared sector these modes must be real in order to $[\chi_{cg}, \dot{\chi}_{cg}] = [h_{cg}, \dot{h}_{cg}] = 0$. The power $h_{cg}^{(c)}$ - spectrum in the infrared sector is given by eq. (54). The modes ξ_k are the solution of the eq. (36) and $\tilde{\xi}_k(t)$ can be calculated by means of (32). In the equation that describes the evolution for χ [see eq. (27)], an effective time - dependent parameter of mass $\mu(t) = \frac{k_o(t)}{a(t)}$ appears.

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